CLASS 10 · MATHEMATICS · CHAPTER 8

Introduction to Trigonometry - Exercise 8.3

Exercise 8.3 with independently verified solutions.

CHAPTER	RESOURCE	SOURCE
Introduction to Trigonometry	NCERT Exercises	NCERT Mathematics, Reprint 2026-27

PART 1

Verified NCERT questions and solutions

Verified NCERT questions and solutions

Solutions are checked independently and follow the supplied NCERT chapter edition.

QUESTION

Question 1

identities · in terms of cot

Express sin A, sec A and tan A in terms of cot A.

Solution

Board-exam working:

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$

$$\operatorname{cosec} A = \sqrt{1 + \cot^2 A}$$

$$\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{1 + \cot^2 A}}$$

$$\tan A = \frac{1}{\cot A}$$

$$\sec^2 A = 1 + \tan^2 A = 1 + \frac{1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}$$

Final answer

$$\sin A = \frac{1}{\sqrt{1 + \cot^2 A}}, \quad \sec A = \frac{\sqrt{1 + \cot^2 A}}{\cot A}, \quad \tan A = \frac{1}{\cot A}$$

QUESTION

Question 2

identities · in terms of sec

Write all other trigonometric ratios of angle A in terms of sec A.

Solution

Board-exam working:

$$\cos A = \frac{1}{\sec A}$$

$$\sec^2 A = 1 + \tan^2 A \Rightarrow \tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\sin A = \tan A \cos A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$$\operatorname{cosec} A = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Final answer

$$\cos A = \frac{1}{\sec A}, \quad \tan A = \sqrt{\sec^2 A - 1}, \quad \cot A = \frac{1}{\sqrt{\sec^2 A - 1}}, \quad \sin A = \frac{\sqrt{\sec^2 A - 1}}{\sec A}, \quad \operatorname{cosec} A =$$

QUESTION

Question 3

multiple choice · identities

Choose and justify: (i) $9\sec^2 A - 9\tan^2 A$; (ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$; (iii) $(\sec A + \tan A)(1 - \sin A)$; (iv) $(1 + \tan^2 A)/(1 + \cot^2 A)$.

Solution

Board-exam working:

(i)

$$9(\sec^2 A - \tan^2 A) = 9$$

Option B.

(ii) Let t = tan θ, so cot θ = 1/t and cosec θ = sec θ/t.

$$(1 + t + \sec\theta)\left(1 + \frac{1}{t} - \frac{\sec\theta}{t}\right)$$

$$= \frac{(1 + t + \sec\theta)(1 + t - \sec\theta)}{t}$$

$$= \frac{(1+t)^2 - \sec^2\theta}{t} = \frac{1 + 2t + t^2 - (1 + t^2)}{t} = 2$$

Option C.

(iii)

$$(\sec A + \tan A)(1 - \sin A) = \frac{1 + \sin A}{\cos A}(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \cos A$$

Option D.

(iv)

$$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \tan^2 A$$

Option D.

Final answer

(i) B; (ii) C; (iii) D; (iv) D.

QUESTION

Question 4

prove identities · ten subparts

Prove the following identities for acute angles where the expressions are defined:

$$\text{(i) } (\operatorname{cosec}\theta - \cot\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$$

$$\text{(ii) } \frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A} = 2\sec A$$

$$\text{(iii) } \frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$$

$$\text{(iv) } \frac{1 + \sec A}{\sec A} = \frac{\sin^2 A}{1 - \cos A}$$

$$\text{(v)}\ \frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$$

$$\text{(vi)}\ \sqrt{\frac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$$

$$\text{(vii)}\ \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$$

$$\text{(viii)}\ (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$$

$$\text{(ix)}\ (\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$$

$$\text{(x)}\ \frac{1 + \tan^2 A}{1 + \cot^2 A} = \left(\frac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$$

Solution

Board-exam working:

(i)

$$(\operatorname{cosec}\theta - \cot\theta)^2 = \left(\frac{1 - \cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1 - \cos\theta)^2}{1 - \cos^2\theta} = \frac{1 - \cos\theta}{1 + \cos\theta}$$

(ii)

$$\frac{\cos A}{1 + \sin A} + \frac{1 + \sin A}{\cos A}$$

$$= \frac{1 - \sin A}{\cos A} + \frac{1 + \sin A}{\cos A} = \frac{2}{\cos A} = 2\sec A$$

(iii)

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)}-\frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$=\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$$

$$=\frac{1+\sin\theta\cos\theta}{\sin\theta\cos\theta}=1+\sec\theta\,\mathrm{cosec}\,\theta$$

(iv) Left-hand side:

$$\frac{1+\sec A}{\sec A}=1+\cos A$$

Right-hand side:

$$\frac{\sin^2 A}{1-\cos A}=\frac{(1-\cos A)(1+\cos A)}{1-\cos A}=1+\cos A$$

(v) Use 1 – cos A = sin²A/(1 + cos A).

$$\cos A+\sin A-1=\sin A-(1-\cos A)$$

$$=\sin A-\frac{\sin^2 A}{1+\cos A}=\frac{\sin A(1+\cos A-\sin A)}{1+\cos A}$$

$$\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\frac{1+\cos A}{\sin A}=\mathrm{cosec}\,A+\cot A$$

(vi)

$$\sqrt{\frac{1+\sin A}{1-\sin A}}=\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} = \sec A + \tan A$$

(vii)

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

$$= \frac{\sin\theta(\cos^2\theta - \sin^2\theta)}{\cos\theta(\cos^2\theta - \sin^2\theta)} = \tan\theta$$

(viii)

$$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= \sin^2 A + 2 + \operatorname{cosec}^2 A + \cos^2 A + 2 + \sec^2 A$$

$$= 5 + (1+\cot^2 A) + (1+\tan^2 A) = 7 + \tan^2 A + \cot^2 A$$

(ix) Left-hand side:

$$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$$

$$= \frac{1-\sin^2 A}{\sin A} \cdot \frac{1-\cos^2 A}{\cos A} = \sin A \cos A$$

Right-hand side:

$$\frac{1}{\tan A + \cot A} = \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \sin A \cos A$$

(x) First expression:

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \tan^2 A$$

Second expression:

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2=\left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$=\left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2=(-\tan A)^2=\tan^2 A$$

Final answer

All ten identities are proved.